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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 907

SOME INVESTIGATIONS OF THE GENERAL INSTABILITY OF STIFFENED METAL CYLINDERS

III - CONTINUATION OF TESTS OF WIRE-BRACED SPECIMENS AND PRELIMINARY TESTS OF SHEET-COVERED SPECIMENS

Guggenheim Aeronautical Laboratory
California Institute of Technology

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This is the third of a series of reports covering an investigation of the general instability problem by the California Institute of Technology. The first five reports of this series cover investigations of the general instability problem under the loading conditions of pure bending and were prepared under the sponsorship of the Civil Aeronautics Administration. The succeeding reports of this series cover the work done on other loading conditions under the sponsorship of the National Advisory Committee for Aeronautics.

INTRODUCTION

This report is concerned primarily with the continuation of the tests of wire-braced specimens (references 1 and 2), and preliminary tests of sheet-covered specimens that had been made in the experimental investigation on the problem of the general instability of stiffened metal cylinders at the C.I.T. Tests have been completed on the first series of specimens. These specimens were constructed, using one size of frame and longitudinal and were given torsional stiffness by a wire network. A discussion of this type of specimen is given in appendix B of the second report of this series (reference 2). In this set of specimens longitudinal spacing was varied from 2.53 inches to 10.12 inches and the frame spacing was varied from 2 inches to 32 inches.

In addition to the previous set of specimens, four cylinders with the same size frame and longitudinal and covered with 0.010-inch-thick 17S-T dural sheet have been tested. A tentative correlation parameter has been found and is discussed more fully in the body of the report.

EXPERIMENTAL INVESTIGATION OF WIRE-BRACED SPECIMENS

Details of the specimens and of the testing procedure are given in reference 2. Table I gives the complete set of specimens which have been tested and gives the number of frames and longitudinals in each specimen and the failing-bending moments. Specimens 1 to 15 were discussed in reference 2, while this report presents the data gathered from the tests of specimens 16 to 24.

Tables II, III, and IV give the section properties and the failing bending moments for the specimens having various frame spacings and 2.53-, 5.06-, and 10.12-inch longitudinal spacings, respectively. These data are plotted in figure 1, showing the failing-bending moment as a function of frame spacing for each of the longitudinal spacings tested. These three curves indicate clearly that the effect of changing the frame spacing is reduced as the frame spacing becomes larger. Since there are no discontinuities or sudden changes of slope in the curves, it is concluded that essentially the same type of failure took place in all of the specimens tested. This was borne out by the observations during the tests, since only one specimen - 21, which had a 5.06-inch longitudinal spacing and a 32-inch frame spacing - showed any tendency toward the panel-type of failure. All other failures were of the general instability type in which both longitudinals and frames showed marked deflections at failure.

An item of interest is the compression stress in the most highly loaded longitudinal when the cylinder is subjected to the failing bending moment. This has been calculated and is shown plotted in figure 2 against the frame spacing. This set of curves indicates that the critical longitudinal stress is not a simple function of the longitudinal spacing, but reaches a maximum for some spacing between 2.53 and 10.12 inches for frame spacings greater than 2 inches. However, at a frame spacing of 2 inches it is seen that the curves cross and the 10.12-inch longitudinal spacing gives the maximum value for the critical stress.

The work of Dschou, which was discussed in reference 1, indicated that the critical stress was a linear function of the parameter $\sqrt{I_s I_f / bd}$. The experimentally determined critical stresses have been plotted in figure 3 against this parameter, and it is immediately evident that a linear relationship does not hold. Figures 2 and 3 indicate that further study is necessary in order to determine the dependence of the critical longitudinal stress on the physical constants of the test specimen.

In order to obtain some measure of the efficiency of the various frame and longitudinal combinations, a moment-weight ratio was calculated for each specimen. The results of this calculation are shown in figure 4, and they indicate that the specimens with 2.53- and 5.06-inch longitudinal spacings have the same structural efficiency; however, the maximum value occurs for different frame spacings in the two series. The series with the 10.12-inch longitudinal spacing falls considerably below the other two curves and it does not appear that specimens with this spacing of longitudinals could ever be made as efficient as those with the smaller longitudinal spacings.

The maximum radial deflections as a function of the applied bending moments are plotted in figures 5 and 6. These two figures are similar to figure 54 (reference 2) on which were plotted the data for specimens with a longitudinal spacing of 2.53 inches. The shape of these curves indicated that it might be possible to apply the Southwell method of determining the asymptotic value of the maximum bending moment. In this method, δ is plotted against δ/M and the inverse slope of the resulting straight line is equal to the horizontal asymptote of the M against the δ curve. This method was tried in several cases and was found to give a value for the horizontal asymptote somewhat higher than that obtained experimentally. In most cases the experimental value of the failing bending moment was between 80 and 90 percent of the value obtained by using the Southwell method.

EXPERIMENTAL INVESTIGATION OF SHEET-COVERED SPECIMENS

Before additional tests on wire-braced specimens were made, it was decided to test a limited number of sheet-covered specimens in order to determine whether a correlation could be obtained between the failing bending moments on the two types of structure. In other words, it was desired to know whether it would be possible to predict the failing bending moment of a sheet-covered specimen from the results obtained in testing a similar specimen without sheet covering, but which was wire-braced.

To date, four specimens having the same type longitudinals and frames as those used in specimens 1 to 24 and covered with 0.010-inch-thick 17S-T dural sheet have been tested. Table V gives the properties and failing bending moments of these specimens and figure 7 shows the variation in failing bending moment with frame spacing as compared to that obtained in the representative wire-braced specimens. It is interesting to note that there is almost a constant multiplying factor between the two critical bending moments on specimens with the same frame spacing. The correct significance of this factor is one of the problems which is now being investigated from the theoretical standpoint.

NORMAL RESTRAINT COEFFICIENT

During the testing of both the wire-braced and the sheet-covered specimens, it was observed that the resistance of the specimen to any externally applied radial load became less and less as the bending moment on the specimen was increased. It was therefore decided to make accurate measurements of this property of the specimen in order to determine whether it might be connected in some manner with the ultimate strength of the structure.

In order to measure this resistance to radial deformation, weights were hung at the center of the lowest (compression) longitudinal, and the radial deflection (δ) was measured at the weight attachment point. This was done with the specimen in the unloaded state and also for various values of increasing bending moment. A typical series of results is shown in figure 8. This family of curves indicates clearly that the resistance to any radial load decreases as the applied bending moment on the structure increases.

Similar sets of curves have been taken for a number of the specimens tested, and the results have all been plotted as shown by a typical curve in figure 9. In this curve, the slopes of the P against the δ curves have been plotted as ordinates and the applied bending moments as abscissas. All of the curves which have been investigated to date show a nearly linear relationship between P/δ and a bending moment up to approximately 70 to 75 percent of the failing bending moment. Beyond this point, the linear relationship no longer holds, the P/δ values falling rapidly as the failing bending moment is approached.

Further investigations showed that the value of P/δ was directly connected with the ultimate load which could be carried by the structure. This is strikingly shown by figure 10 in which the failing bending moment for a number of specimens has been plotted against the respective P/δ values, the specimens being in the unloaded state. As is indicated in this figure, six wire-braced and four sheet-covered specimens have been tested and no specimen varies more than 8 percent from the mean straight line. Table VI gives the details of the specimens considered in figure 10.

It is felt that the discovery of the relationship indicated in figure 10 may greatly affect future methods of analysis and design of structures subject to a general instability type of failure. The first item of importance is the fact that the radial stiffness of a stiffened cylinder is in some way a measure of the ultimate bending moment which such a structure can support. Secondly, it may be possible to test simple wire-braced specimens and extrapolate the results to more expensive sheet-covered structures. Furthermore, if the results of figure 10 are found to be general in nature, the tests can be made without applying any load to the structure other than those loads necessary to obtain the P/δ values.

If the above principles are substantiated by further investigation, testing of existing structures will be comparatively easy; however, the results as yet yield no method of calculating the allowable bending moment for proposed designs. For this purpose, it will be necessary to determine the parameters which fix the value of the restraint coefficient for any structure.

All specimens so far tested have failed by a general instability type of buckling. For these specimens, it appears that it is immaterial whether the normal restraint coefficient is measured at a frame-longitudinal joint or at some point on the longitudinal between two frames. It is believed that this will not be the case in structures which are subject to a panel-instability type of failure, and such a result may lead to a method of determining the type of failure to which a particular structure is subject.

Guggenheim Aeronautical Laboratory,
California Institute of Technology,
Pasadena, Calif., July 1939.

REFERENCES

1. Guggenheim Aeronautical Laboratory, California Institute of Technology: Some Investigations of the General Instability of Stiffened Metal Cylinders. I - Review of Theory and Bibliography. T.N. No. 905, NACA, 1943.

2. Guggenheim Aeronautical Laboratory, California Institute of Technology: Some Investigations of the General Instability of Stiffened Metal Cylinders. II - Preliminary Tests of Wire-Braced Specimens and Theoretical Studies. T.N. No. 906, NACA, 1943.

Table I.

Experimental Data - Set I

Pure Bending Tests of Longitudinal - Frame Combinations

Without Sheet Covering

All Longitudinals S_1 All Frames F_5

Specimen No.	Longitudinal Spacing	Longitudinal No.	Frame Spacing	Frame No.	Failing Moment - lbs.	Description of Test	Remarks
1	2.53"	40	4"	15	33,300	No wire bracing	Longitudinals failed in circumferential direction
2	"	"	"	"	90,000	Wire bracing	Longitudinals failed in radial direction 1/2 wave complete length of specimen.
3	"	"	"	"	87,000	Wire bracing	General instability same as specimen No. 2.
4	"	"	8"	7	60,800	Wire bracing	Gen. inst. 1/2 wave complete length of specimen.
5	"	"	16"	3	49,000	Wire bracing	Gen. inst., however, specimen looked like combination of small waves between frames and one long 1/2 wave.
6	"	"	32"	2	40,000	Wire bracing	Similar to specimen 5. General instability.
7	"	"	32"	1	36,300	Wire bracing	Specimens started to fail between frames, however, as deflection increased, frame failed.
8	"	"	64"*	0	18,000	Wire bracing	Long. failed in one long 1/2 wave in rad. direction.
9	"	"	2"	31	60,000	No wire bracing	Longitudinals failed in circumferential direction.
10	"	"	2"	31	90,000	Some wire bracing	Bottom longitudinal failed partly in radial direction and partly in circumferential.
11	"	"	2"	31	102,000	Wire bracing	Gen. inst., one long 1/2 wave. Bottom longitudinal failed little in circumferential direction.
12	"	"	2"	31	114,000	Wire bracing	General instability, one long half wave longitudinals failed in radial direction.
13	5.06"	20	2"	31	65,000	Wire bracing	Gen. inst., length of buckle was less than length of specimen, about 25 frames failed.

*Total length of specimen, distance between supports.

Table I cont.

Experimental Data - Set I

Specimen No.	Longitudinal Spacing	Longitudinal No.	Frame Spacing	Frame No.	Failing Moment - lbs.	Description of Test	Remarks
14	10.12A	10	2"	31	33,000	Wire bracing	Gen. inst., one to one and half complete waves in length-wise direction.
15	10.12B	10	2"	31	31,500	Wire bracing	General instability, one and half waves in length-wise direction.
16	10.12A	10	4"	16	26,500	Wire bracing 4" Frame x 2.5" Stiff.	General instability, one and half waves in length-wise direction.
17	10.12B	10	4"	16	25,500	Wire bracing	General instability. Several waves.
18	5.06	20	4"	16	55,000	Wire bracing 4"F x 2.5" Stiff.	General instability. 1/2 wave.
19	5.06	20	8"	7	43,800	5"F x 8" Stiff.	
20	5.06	20	16"	3	31,000	Wire bracing 5"F x 8" Stiff.	
21	5.06	20	32"	2	22,300	5"F x 8" Stiff.	Panel instability. Only slight deformation of frames.
22	5.06	20	32"	1	21,000		
23	10.12A	10	8"	7	19,000	5"F x 8" Stiff.	General instability. 1/2 wave.
24	10.12A	10	16"	3	14,500	10"F x 16" Stiff.	General instability. 1/2 wave.

A - Single longitudinal at maximum stress. B - Two longitudinals equal distance from

Table II.

The Effect of Changing the Frame Spacing on the Bending Strength of the Cylinders

Spec. No.	Longitudinal Type	Area A_s in²	Mom. of inertia $I_s \times 10^4$ in⁴	Spac. b ins	$\frac{I_s \times 10^4}{b}$	Frame Type	Area A_f in²	Mom. of inertia $I_f \times 10^5$	Spac. d ins	No. of frames in 64"	$\frac{I_f}{d} \times 10^6$	Failing Moment B.M. in-lbs.	Maximum Compr. stress at Failure σ_{cr}	Type of Failure
12	S_1	.0324	3.74	2.53	1.478	F_5	.0291	1.537	2	31	7.690	114,000	11,160	General instability
2	"	"	"	"	"	"	"	"	4	15	3.845	90,000	8,810	" "
3	"	"	"	"	"	"	"	"	4	15	"	87,000	8,500	" "
4	"	"	"	"	"	"	"	"	8	7	1.923	60,800	5,950	" "
5	"	"	"	"	"	"	"	"	16	3	0.962	49,000	4,800	Nearly panel instability
6	"	"	"	"	"	"	"	"	32	2	0.481	40,000	3,910	Nearly panel instability
7	"	"	"	"	"	"	"	"	32	1	0.481	36,300	3,555	General instability

Moment of inertia of complete specimen = I_{sp} = 161.0 in⁴

Radius to center of stiffener R = 15.76"

Number of stiffeners = 40

$$\sigma_{cr} = \frac{BM \times R}{I_{sp}} = \frac{BM \times 15.76}{161.0} = .0979 \times BM$$

Length of specimen = 64".

Table III.

The Effect of Changing the Frame Spacing on the Bending Strength of Cylinders

Longitudinal Spacing = 5.06"

Spec. No.	Longitudinal					Frame						Failing Moment B.M. in-lbs.	Maximum Compr. stress at Failure σ_{cr}
	Type	Area A_s in^2	Mom. of inertia $I_s \times 10^4$ in^4	Spac. b ins	$\frac{I_s}{b} \times 10^4$	Type	Area A_f in^2	Mom. of inertia $I_f \times 10^5$	Spac. d ins	No. of frames in 64"	$\frac{I_f}{d} \times 10^6$		
13	S_1	.0324	3.74	5.06	.739	F_5	.0291	1.537	2"	31	7.690	65,000	12,729
18	"	"	"	"	"	"	"	"	4"	16	3.845	55,000	10,771
19	"	"	"	"	"	"	"	"	8"	7	1.923	43,800	8,577
20	"	"	"	"	"	"	"	"	16"	3	0.962	31,000	6,071
21	"	"	"	"	"	"	"	"	32"	2	0.481	22,300	4,367
22	"	"	"	"	"	"	"	"	32"	1	0.481	21,000	4,112

Moment of inertia of complete specimen = I_{sp} = 80.48 in.^{4}

Radius to center of longitudinals R = 15.76"

Number of longitudinals = 20

Length of specimen = 64"

Table IV.

The Effect of Changing the Frame Spacing on the Bending Strength of Cylinders

Longitudinal Spacing = 10.12"

Spec. No.	Longitudinal					Frame						Failing Moment B.M. in-lbs.	Maximum Compr. stress at Failure σ_{cr}
	Type	Area A_s in^2	Mom. of inertia I_sx10^{4}in^4	Spac. b ins	$\frac{I_s}{b}$ x10^4	Type	Area A_f in^2	Mom. of inertia I_fx10^5	Spac. d ins	No. of frames in 64"	$\frac{I_f}{d}$ x10^6		
14	S_1	.0324	3.74	10.12A	.370	F_5	.0291	1.537	2"	31	7.690	33,000	12,926$_A$
15	"	"	"	10.12B	"	"	"	"	2"	31	7.690	31,500	9,406$_B$
16	"	"	"	10.12A	"	"	"	"	4"	16	3.845	26,500	10,380$_A$
17	"	"	"	10.12B	"	"	"	"	4"	16	3.845	25,500	7,614$_B$
23	"	"	"	10.12A	"	"	"	"	8"	7	1.923	19,000	7,442$_A$
24	"	"	"	10.12A	"	"	"	"	16"	3	0.962	14,500	5,680$_A$

A - Single longitudinal at maximum stress.

A -- Moment of inertia of complete specimen = I_{sp} = 40.24 in.^{4}

Y = R (Distance to extreme fiber.)

B - Two longitudinals equal distance from ℄

B - Moment of inertia of complete specimen = I_{sp} = 50.19 in.^{4}

Y = 15.0"

Radius to center of longitudinals = R = 15.76"

Number of longitudinals = 10

Length of specimen = 64"

Table V.

Experimental Data - Set 2

Pure Bending Tests of Longitudinal - Frame Combinations

Without Sheet Covering

Spec. No.	Longitudinal				Frame				Sheet Thickness t ins.	Failing Moment $B.M._F$ in-lbs.	Type of Failure
	Type	Area A_s in^2	Mom. of inertia $I_s \times 10^4$ in^4	Spac. b ins.	Type	Area A_f in^2	Mom. of inertia $I_f \times 10^5$	Spac. d ins.			
25	S_1	.0324	3.74	2.63	F_5	.0291	1.537	8	.010	220,000	General instability 1/2 wave in length of spec.
26	"	"	"	"	"	"	"	4	"	274,000	General instability 1 wave in length of spec.
27	"	"	"	"	"	"	"	2	"	359,000	General instability 1 wave in length of spec.
28	"	"	"	"	"	"	"	16	"	168,500	General instability Near panel type.

All material 17S-T

Radius to center of longitudinal = 15.76"

Number of longitudinal = 40

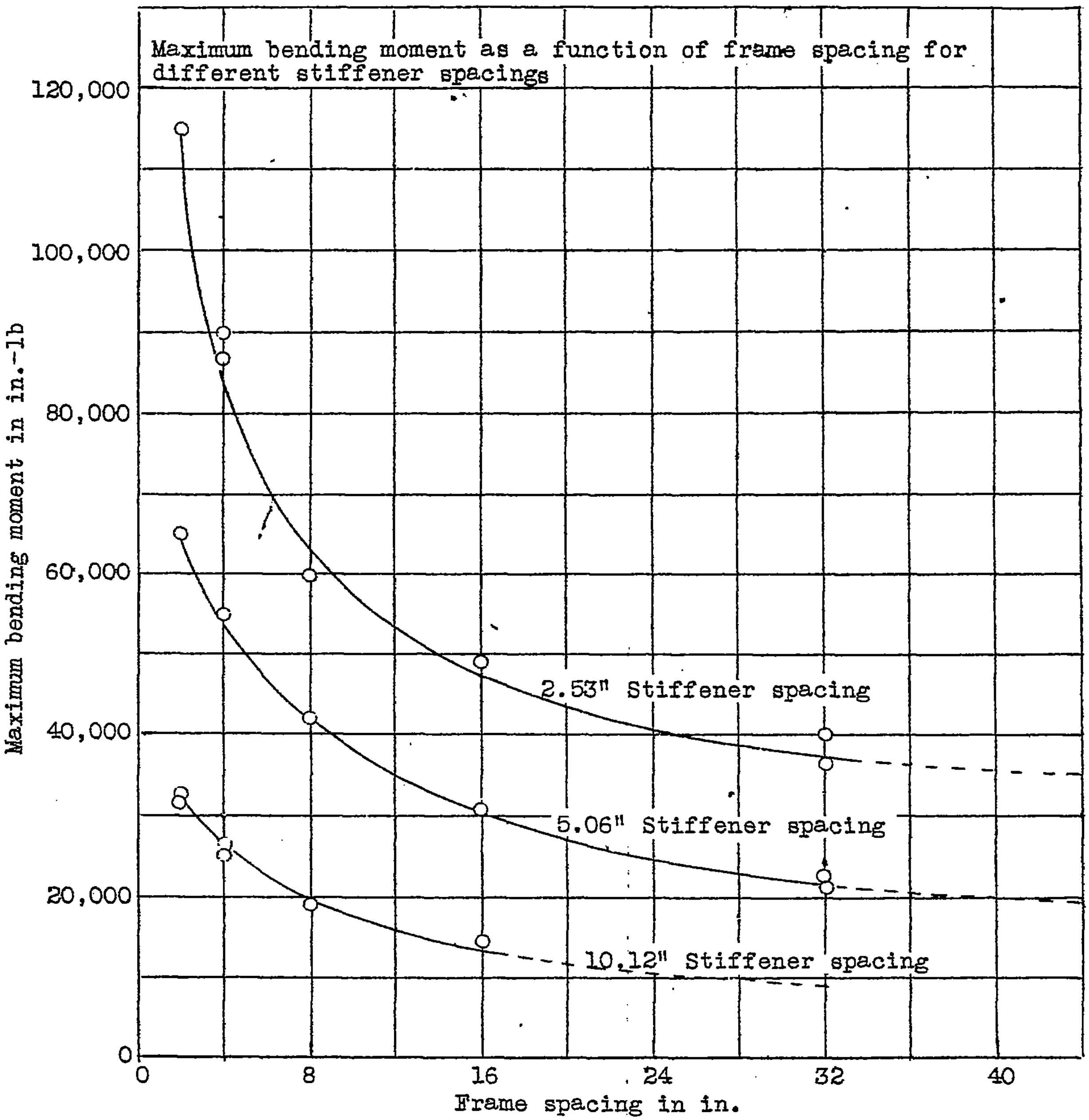
Length of specimen = 64"

TABLE VI

FAILING MOMENT AND NORMAL RESTRAINT COEFFICIENT

[Radius of specimen, 15.76 in.; Length of specimen, 64 in.]

Specimen	Frame spacing (in.)	Longitudinal spacing (in.)	Normal restraint coefficient (for L=64 in.)	Failing moment B.M.F. (in.-lb)	Remarks
12	2	2.53	168.0	114,000	No sheet wire-braced
2	4	2.53	118.2	88,500	Do.
18	4	5.06	70.0	55,000	Do.
19	8	5.06	41.4	43,800	Do.
20	16	5.06	29.3	31,000	Do.
21	32	5.06	12.2	22,300	Do.
28	16	2.53	255.0	168,500	0.010 in. sheet-covered
25	8	2.53	328.9	220,000	Do.
26	4	2.53	452.4	274,000	Do.
27	2	2.53	614.0	359,000	Do.



Maximum bending moment as a function of frame spacing for different stiffener spacings
Maximum bending moment in in.-lb
120,000
100,000
80,000
60,000
40,000
20,000
0
0
8
16
24
32
40
Frame spacing in in.
2.53" Stiffener spacing
5.06" Stiffener spacing
10.12" Stiffener spacing

Figure 1.-

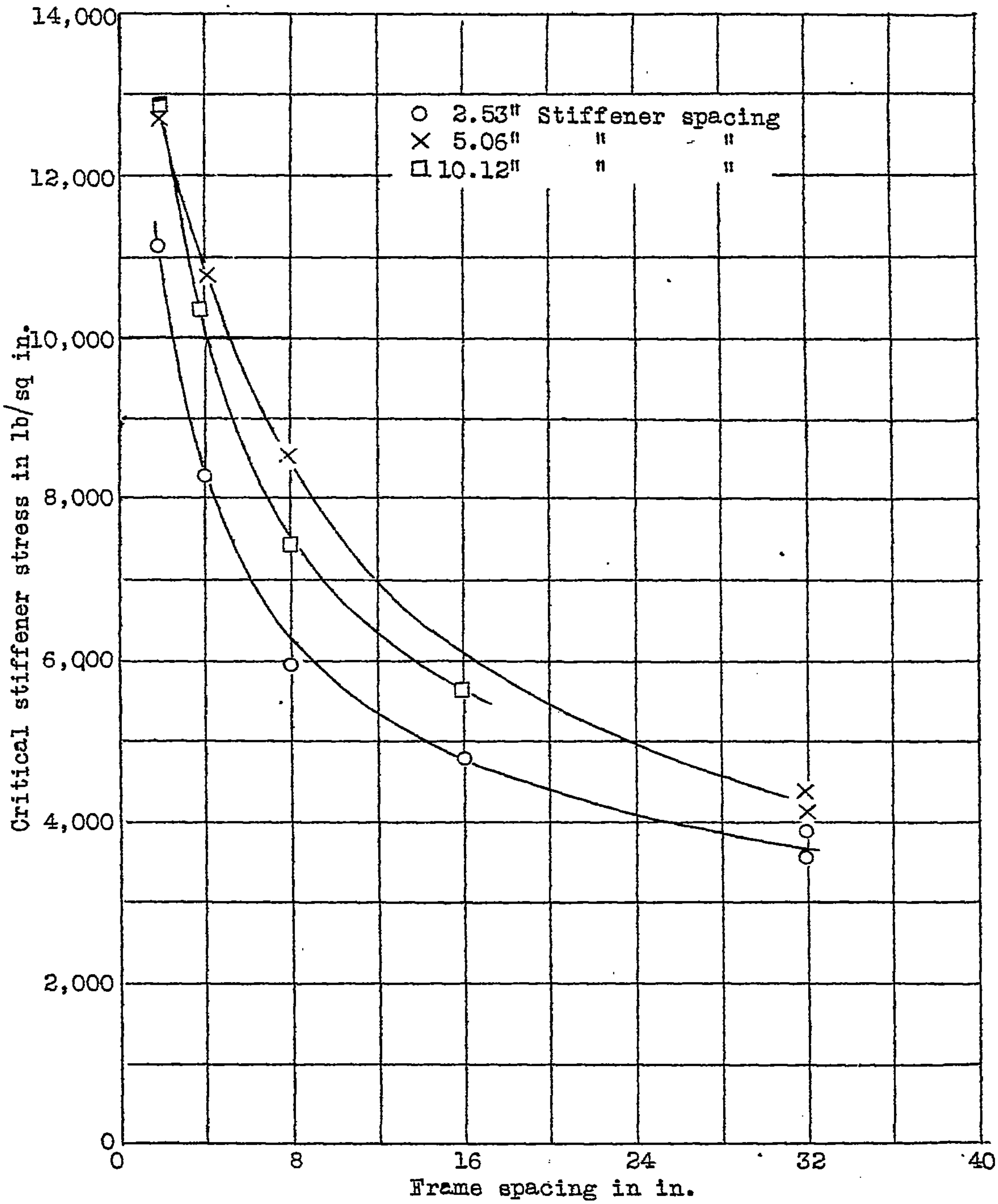

14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
0
8
16
24
32
40
○ 2.53" Stiffener spacing
× 5.06" " "
□ 10.12" " "
Critical stiffener stress in lb/sq in.
Frame spacing in in.

Figure 2.-

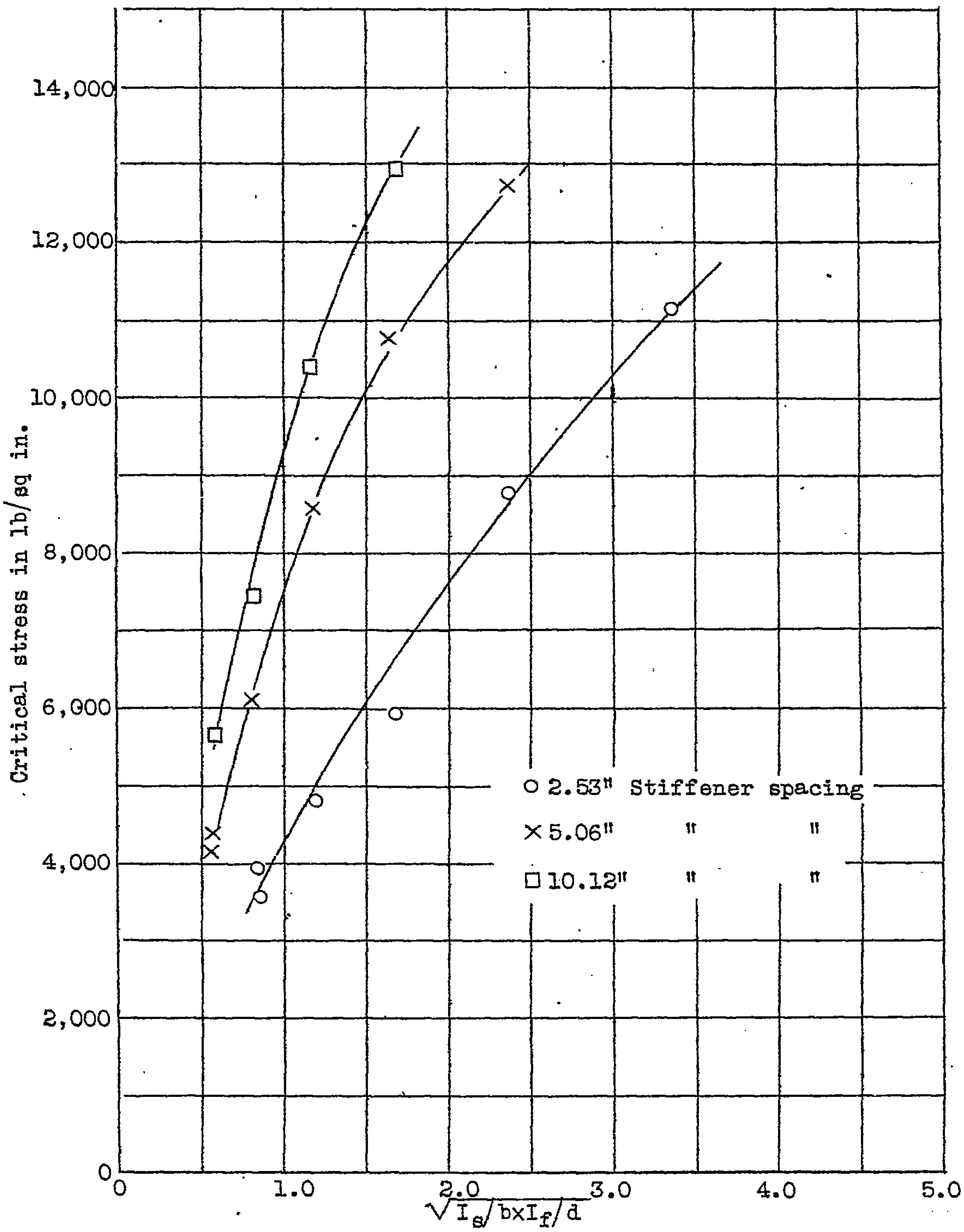

14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
Critical stress in lb/sq in.
0
1.0
2.0
3.0
4.0
5.0
$\sqrt{I_s/bxI_f/d}$
○ 2.53" Stiffener spacing
× 5.06" " "
□ 10.12" " "

Figure 3.-

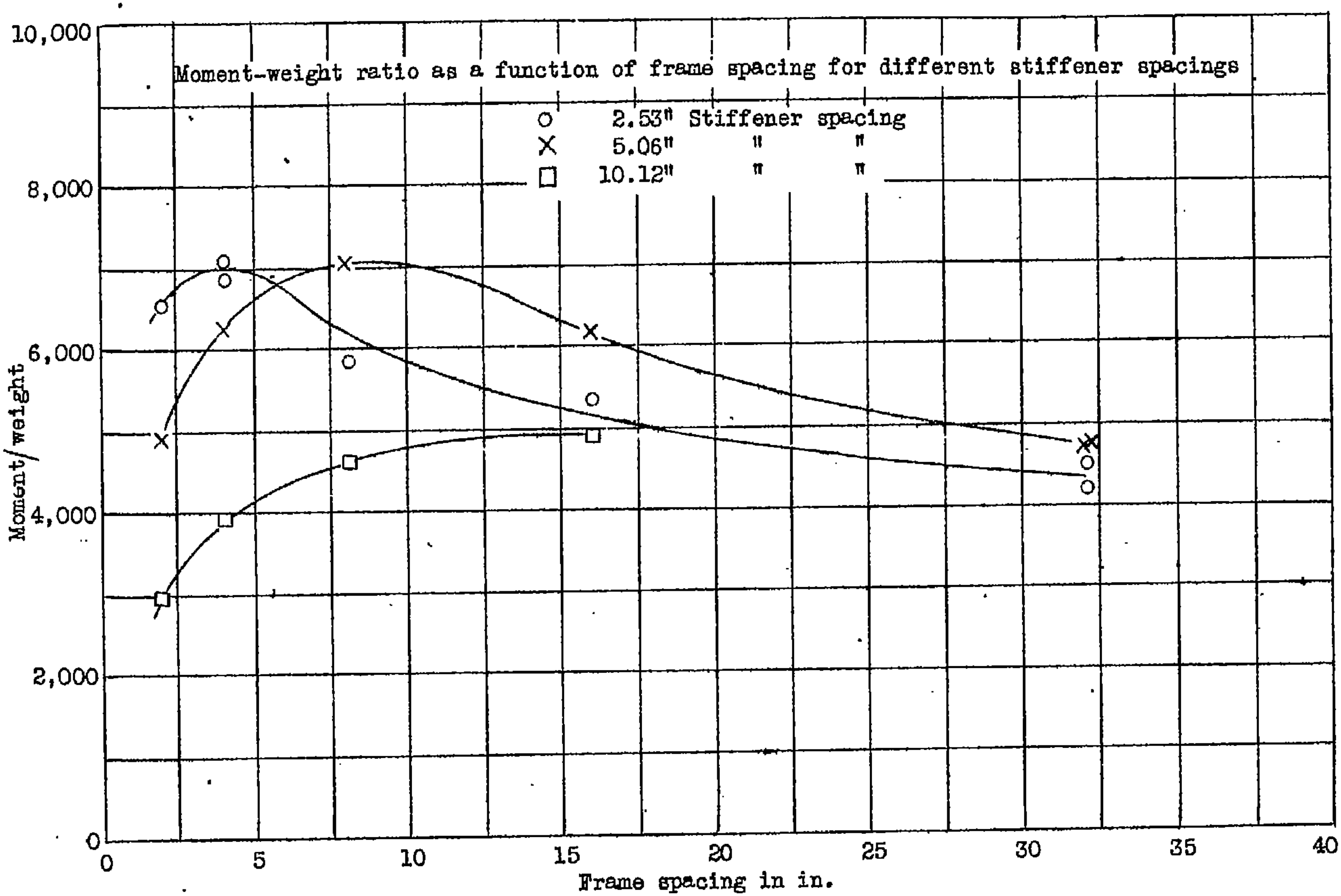

Moment-weight ratio as a function of frame spacing for different stiffener spacings
○ 2.53" Stiffener spacing
× 5.06" " "
□ 10.12" " "
10,000
8,000
6,000
4,000
2,000
0
Moment/weight
0
5
10
15
20
25
30
35
40
Frame spacing in in.

Figure 4.-

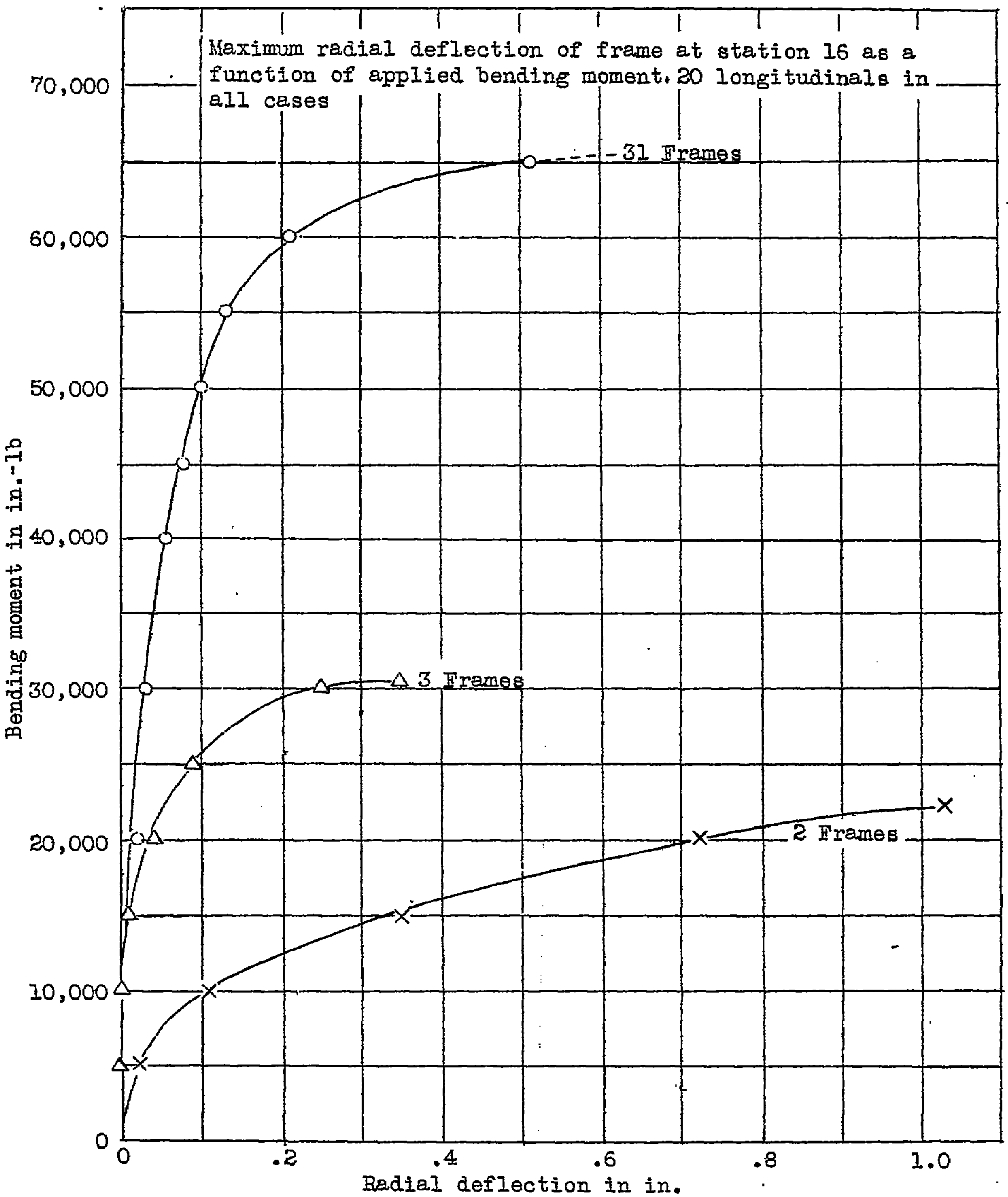

Maximum radial deflection of frame at station 16 as a function of applied bending moment. 20 longitudinals in all cases
31 Frames
3 Frames
2 Frames
Bending moment in in.-lb
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
0
.2
.4
.6
.8
1.0
Radial deflection in in.

Figure 5.-

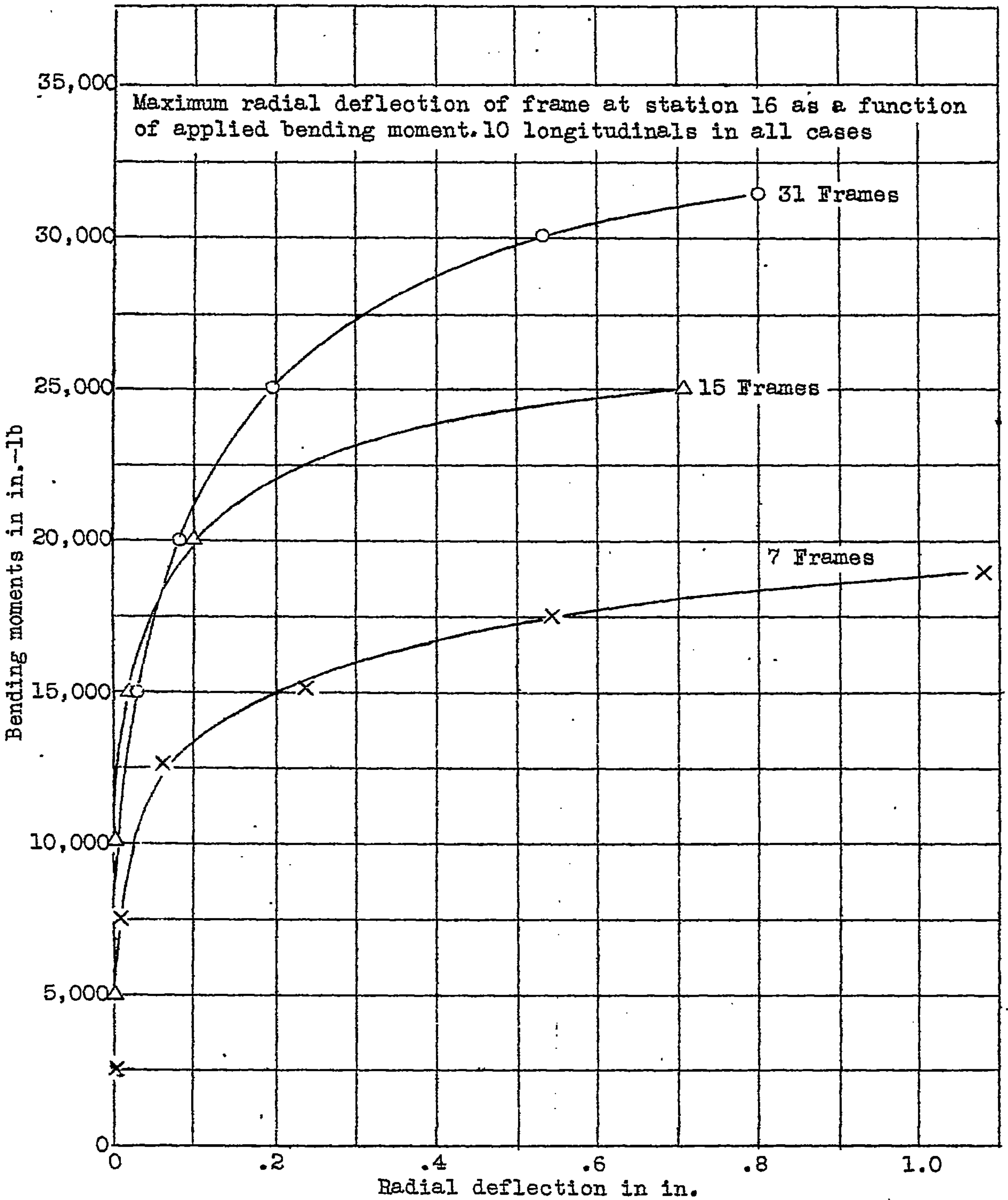

Maximum radial deflection of frame at station 16 as a function of applied bending moment. 10 longitudinals in all cases
31 Frames
15 Frames
7 Frames
Bending moments in in.-lb
Radial deflection in in.
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
0
.2
.4
.6
.8
1.0

Figure 6.-

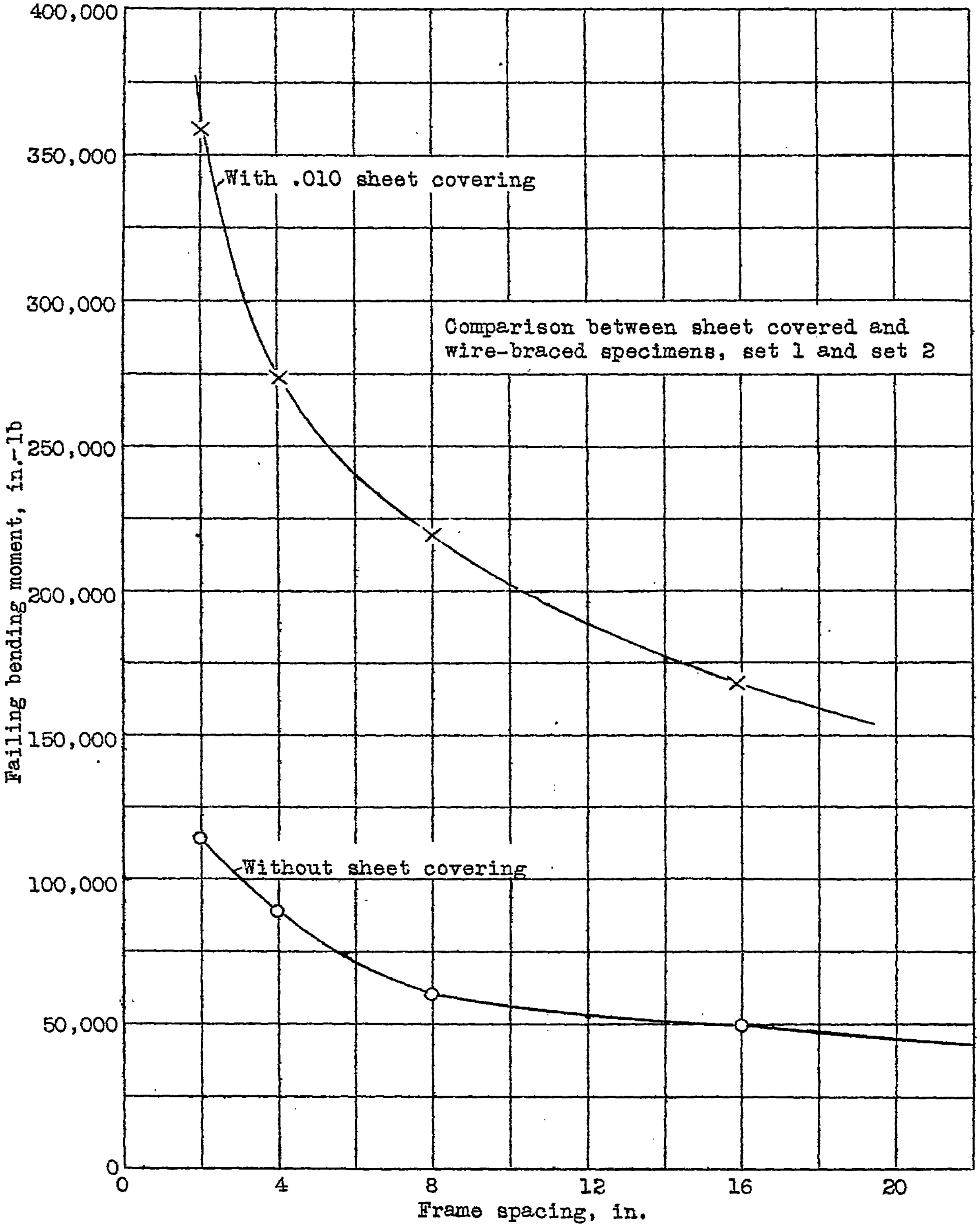

400,000
350,000
300,000
250,000
200,000
150,000
100,000
50,000
0
Failing bending moment, in.-lb
0
4
8
12
16
20
Frame spacing, in.
With .010 sheet covering
Comparison between sheet covered and wire-braced specimens, set 1 and set 2
Without sheet covering

Figure 7.-

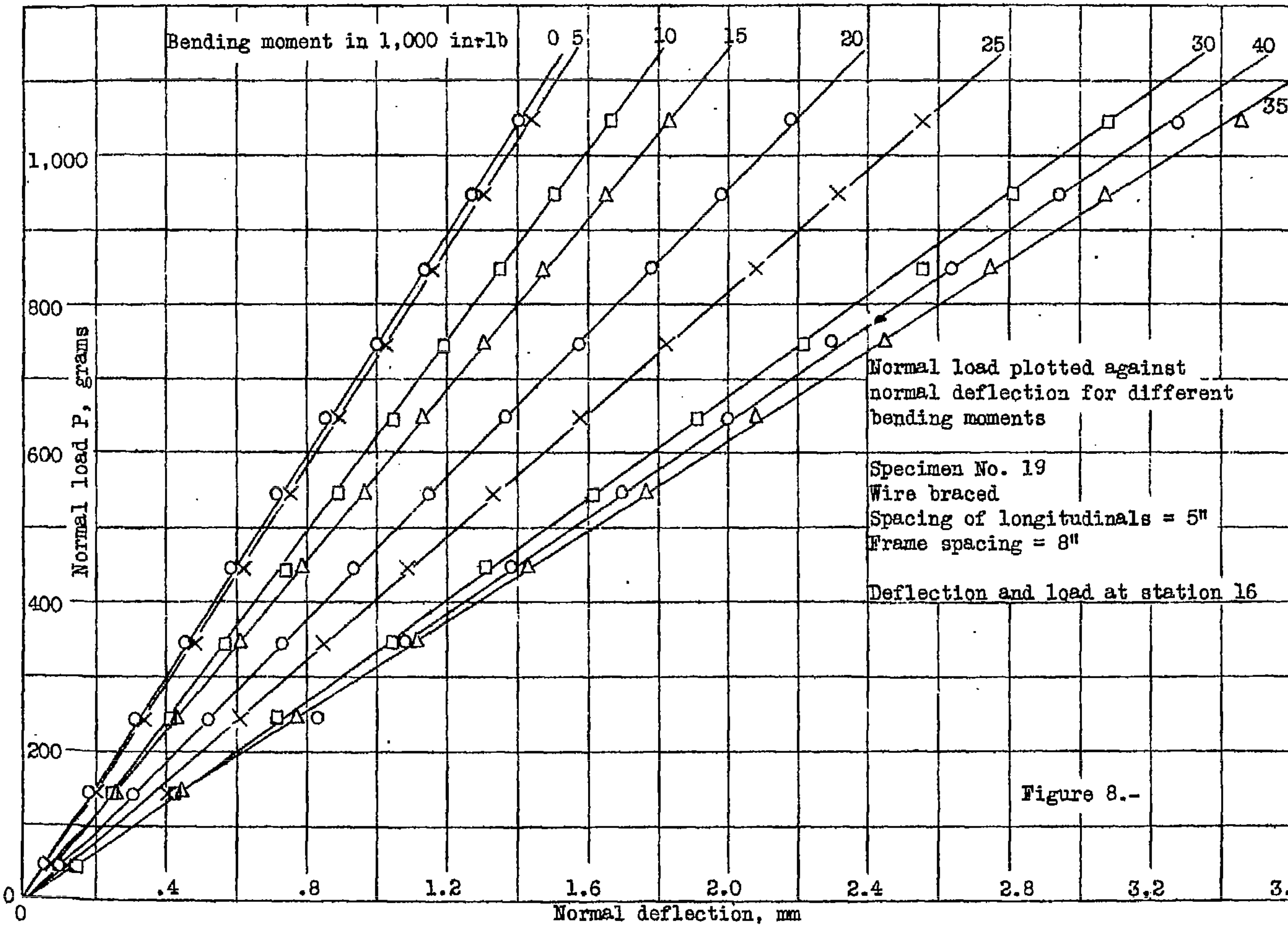

Bending moment in 1,000 in-lb
0
5
10
15
20
25
30
40
35
1,000
800
600
400
200
0
Normal load P, grams
0
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
Normal deflection, mm
Normal load plotted against normal deflection for different bending moments
Specimen No. 19
Wire braced
Spacing of longitudinals = 5"
Frame spacing = 8"
Deflection and load at station 16

Figure 8.-

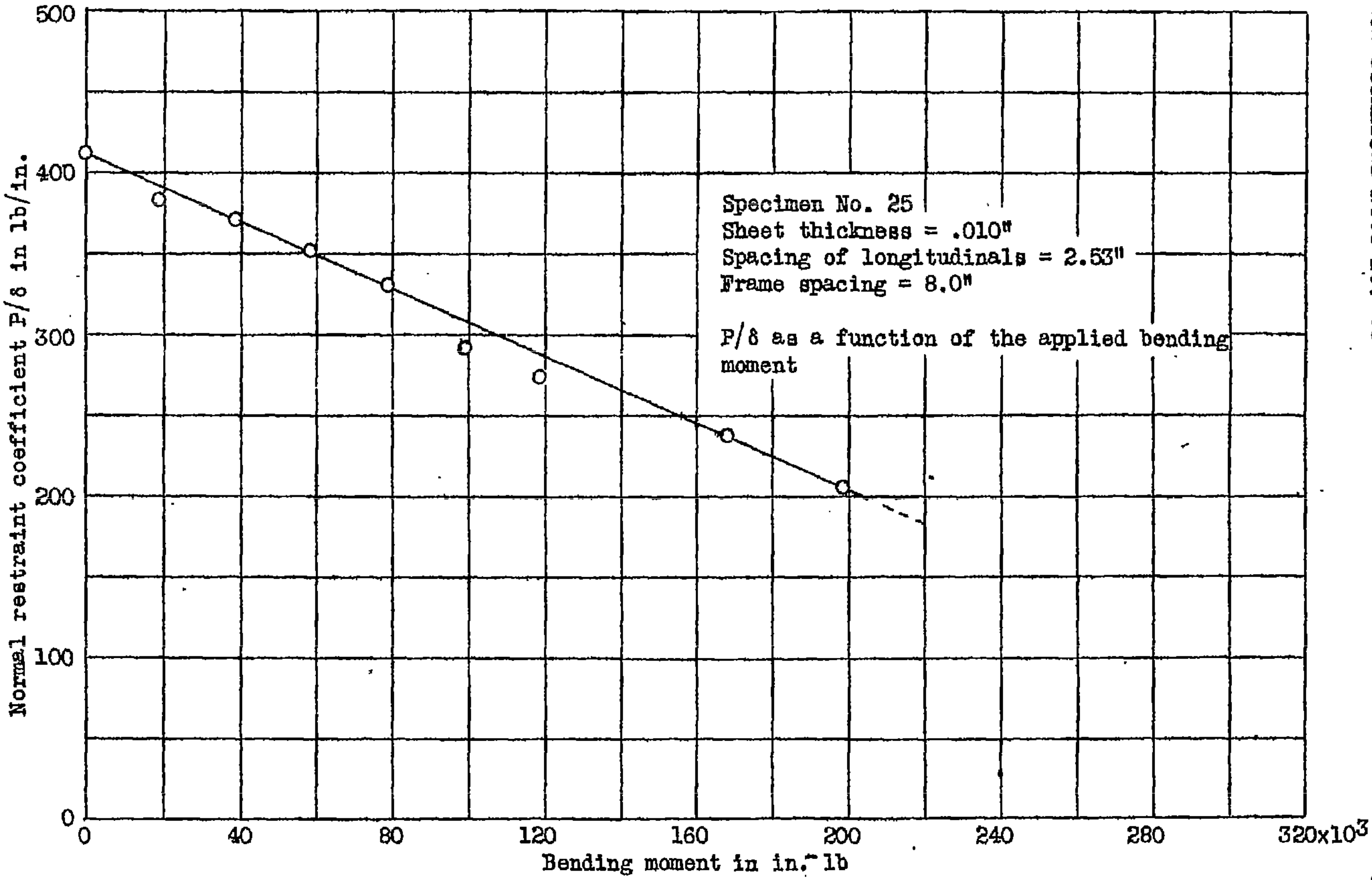

500
400
300
200
100
0
Normal restraint coefficient P/δ in lb/in.
0
40
80
120
160
200
240
280
320x10^3
Bending moment in in.-lb
Specimen No. 25
Sheet thickness = .010"
Spacing of longitudinals = 2.53"
Frame spacing = 8.0"
P/δ as a function of the applied bending moment

Figure 9.-

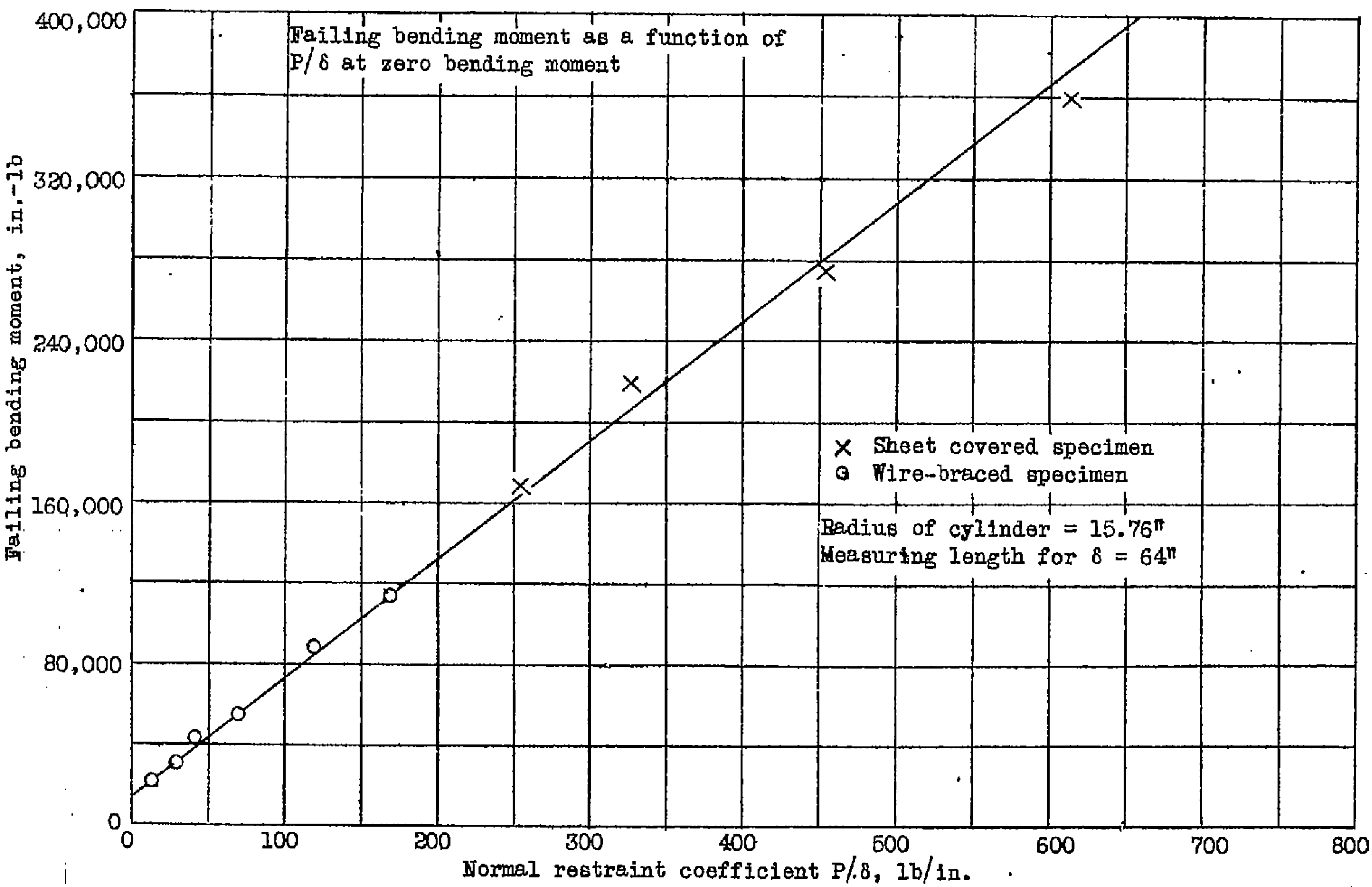

Failing bending moment as a function of
P/δ at zero bending moment
Sheet covered specimen
Wire-braced specimen
Radius of cylinder = 15.76"
Measuring length for δ = 64"
Failing bending moment, in.-lb
400,000
320,000
240,000
160,000
80,000
0
100
200
300
400
500
600
700
800
Normal restraint coefficient P/δ, lb/in.

Figure 10.-